MIDTERM STUDY GUIDE - MATH 100A - FALL 2017

Remark: The list of topics below is not exhaustive. It is likely that there will be things on the exam which do not appear in this study guide.

Know the definition of ...

(1) ... (a, b) the *greatest common divisor of 2 integers.*
(2) ... multiplication of complex numbers, and multiplication of complex numbers in polar form.
(3) ... a *group.*
(4) ... a *subgroup*, $H \leq G$.
(5) ... S_n, the *nth symmetric group.*
(6) ... $A(S)$, the *group of bijections of a set.*
(7) ... D_{2n}, the *dihedral group of order 2n.*
(8) ... $\mathbb{Z}_n$, the *integers modulo n.*
(9) ... $Z(G)$, the *center of a group.*
(10) ... U_n, the *group of integers with multiplicative inverses modulo n.*
(11) ... $C(x)$, the *centralizer of an element* $x \in G$.
(12) ... $o(x)$, the *order of an element* $x \in G$.
(13) ... $\langle x \rangle$, the *subgroup generated by an element* $x \in G$.
(14) ... $\langle S \rangle$, the *subgroup generated by a subset* $S \subset G$.
(15) ... *an equivalence relation* $\sim$ on a set S.
(16) ... *an equivalence class* of an equivalence relation $\sim$.
(17) ... *congruence modulo n.*
(18) ... *a left coset* aH of a subgroup $H \leq G$.
(19) ... *a right coset* Ha of a subgroup $H \leq G$.
(20) ... *a homomorphism* $\varphi \colon G \to H$.
(21) ... $\mathrm{Ker}(\varphi)$, the *kernel of a homomorphism* φ.
(22) ... $\mathrm{Im}(\varphi)$, the *image of a homomorphism* φ.
(23) ... $N \trianglelefteq G$, a *normal subgroup of* G.

For every subgroup in the list of definitions above, you should know how to prove it's a subgroup.

Know the statement and or proof of the following:

(1) Euclid's algorithm for g.c.ds.
(2) $(a, b) = ma + nb$ for some $m, n \in \mathbb{Z}$.
(3) A nonempty subset $S \subset G$ is a subgroup $\iff$ S is closed under $*$, and for all $x \in S$, $x^{-1} \in S$.
(4) $Z(D_{10}) = \{e\}$.
(5) If G is finite then for all $x \in G$ there exists a positive integer n such that $x^n = e$.
(6) If $H, K \leq G$ are subgroups of G, prove that $H \cap K$ is a subgroup of G.
(7) If $H \leq G$, then $x \sim y \iff xy^{-1} \in H$ is an equivalence relation on G.
(8) For the above equivalence relation the equivalence classes are $[a] = Ha$, i.e. the right cosets.
(9) Lagrange's Theorem.
(10) If $a, n \in \mathbb{Z}$ and $(a, n) = 1$ then $a^{\varphi(n)} \equiv 1(\text{mod n})$.
(11) Let $a, p \in \mathbb{Z}$. If p is a prime and $p \nmid a$, then $a^{p-1} \equiv 1(\text{mod p})$.
(12) There is an injective homomorphism $\varphi \colon S_n \to \mathrm{GL}_n(\mathbb{R})$.
(13) Cayley's Theorem.